A Simple Analytic Design Procedure for Lattice Wave Digital Filters with Approximate Linear Phase

Dr. Jassim M. Abdul-Jabbar
Computer Eng. Dept. - University of Mosul, Mosul, Iraq.

Abstract:

A simple analytic design procedure for Lattice Wave Digital Filters (LWDFs) is presented with approximate linear phase. The design is started by replacing one of the two all-pass filter branches in LWDF with a pure delay and terminated by some analytic design formulas. Using Matlab 7.4, several design examples of the odd order type utilizing such procedure are given for verifications.

Keywords: Lattice Wave Digital Filters (LWDFs), All-pass sections, Approximate linear phase.

أسلوب تحليلي مبسط لتصميم مرشحات رقمية موجية من النوع المتشابك وبطور خطي تقريبي

د.جاسم محمد عبد الجبار
قسم هندسة ألحاسبات - جامعة الموصل

الخلاصة-

قدم هذا البحث أسلوبا تحليليا مبسطا لتصميم مرشحات رقمية موجية من النوع المتشابك (LWDFs) وبطور خطي. لقد تم تصميم هذه المرشحات باستخدام تجزئة هذه المرشحات إلى فرعين متوازيين يحتوي ألأول منها على مقاطع إمرار كلية (all-pass sections) بينما يحتوي ألفرع ألثاني على وحدة تأخير خالصة (a pure delay). أمكن الحصول على استجابات خطية ألطور تقريبية لهذه المرشحات و بتصميم مبسط برتب فردية و باستخدام Matlab 7.4 وكما موضح في ألأمثلة.

1. Introduction

A Wave Digital Filter (WDF) is the digital image of a corresponding analogue filter in the analogue reference domain. That is why the design of WDFs can basically be carried out in the analogue domain using classical filter approximations followed by the application of certain analogue to digital transformations rules [1]. Among all other types of recursive filters, WDFs have the advantage of maintaining excellent stability properties for wide-range coefficient variations that take place under finite arithmetic conditions. That because their structures are guaranteed to be free from parasitic oscillations, have a close to minimum coefficient sensitivity, high dynamic range, and good computational properties [2 - 5]. Unfortunately, suboptimal design method of these WDFs results in very high complexity implementations. Particularly, favorable wave digital filters are the Lattice Wave Digital Filters (LWDFs). Using LWDFs, highly modular, perfectly regular and parallel filter structures can be obtained [6]. They are suitable for VLSI implementations as they have regular low complexity structures, low coefficients sensitivity, and can yield optimal pipelining for bit-serial implementations of maximally fast speeds [7 - 9]. Some efficient pipelined WDFs are widely used in wideband high-pass applications such as wireless codec design or electrocardiogram (ECG) signal processing [10], while the others guarantees that the optimum finite-word-length solution can be found for both the fixed-point and the multiplierless coefficient representations [11]. Wave digital realizations can also be obtained from the specifications, through Very High Speed Integrated Circuits (VHSICs) Hardware Description Language (VHDL) and then synthesized into Xilinx Field Programmable Gate Arrays (FPGA) implementations [12]. In addition to that, LWDF is well suited for microcontrollers without a hardware multiplier [13].

Some electronic applications are: 1) high-speed integrated circuits based on the LWDF structures are obtained, 2) multirate IF filters for mobile radio using LWDFs are implemented in silicon, and 3) simple glass breakage detectors are designed using the MSP430 [2, 5, 14]. Other application areas are pulse shaping, audio / image processing systems, digital camera, and mobile phone [4, 15]. Recently,wavelet transform implementations and wavelet bases are obtained from orthonormal nonseparable perfect reconstruction quadrature mirror filter (QMF) banks that are realized either with WDFs or LWDFs [16].

The first design of LWDFs was reported initially by Gazsi in 1985 [17]. Gazsi used some explicit formulas for the direct computation of the adaptor coefficients starting from the poles of the transfer function of the analogue filter predesigned by classical filter approximation techniques. Such design uses the alternative pole technique. LWDFs are inherently branching filters and their realizations are composed of two parallel allpass filter sections [18]. However, such LWDFs can only satisfy some magnitude response requirements without taking any phase response requirement into considerations [12, 19]. Most attempts to design such LWDFs satisfying both magnitude and phase requirements face the problem of no closed form solutions existence. For those attempts, numerical optimization techniques must be adopted [1, 20]. The idea of LWDFs is then extended to the design of almost linear phase LWDFs by replacing one of the two parallel allpass sections by a pure delay [21 , 22]. In all design methods for such LWDFs with approximate phase linearity, there still no existence of any simple analytic design approach.

In this paper, a simple analytic design procedure for LWDFs with approximate linear phase is presented. The procedure is based on the prescribed idea of letting one of the two parallel all-pass filter sections be a pure delay to result in an odd order almost linear phase LWDF. Section 2 of this paper presents the basic ideas of LWDFs. LWDFs with almost linear phase are described in section 3. The design procedure is presented in section 4. Section 5 contains several design examples. Finally, section 6 concludes this paper.

2. Lattice Wave Digital Filters

A LWDF is a two-branch structure, as shown in Fig. 1, where each branch realizes an all-pass filter [17, 18]. These all-pass filters can be realized in several ways. One approach that yields parallel and modular filter algorithms is to use cascaded first- and second-order sections. The first- and second-order sections can be realized as shown in Fig. 2, using symmetric two-port adaptors [1, 21]. Two-port series or parallel adaptors using certain equivalence transformations can easily replace these sections. The second-order sections can also be realized using three-port series or parallel adaptors [21]. Another approach is to realize the all-pass filters using Richard's structure [1], where a processing element can easily be formed to accomplish a bit-serial low-power implementation with low-complexity.

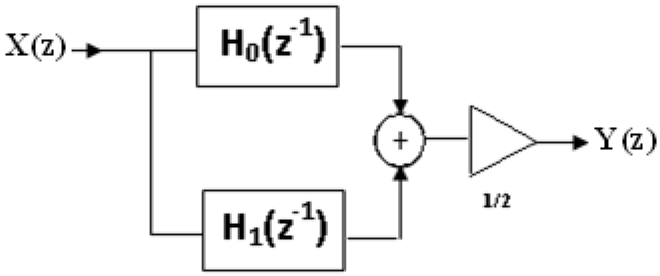


Fig.1 Lattice wave digital filter block diagram.

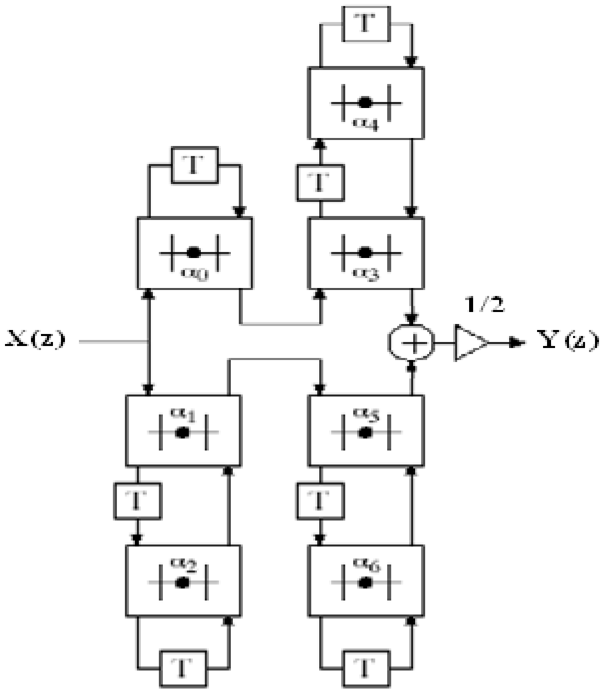


Fig. 2 A 7th order lattice wave digital filter.

The transfer function of a LWDF can be written as

$$H(z) = \frac{1}{2} [H_0(z) + H_1(z)] \qquad (1)$$

where $H_0(z)$ and $H_1(z)$ are all-pass filters. The overall frequency response can therefore, be written as

$$H(e^{j\omega T}) = \frac{1}{2} [e^{j\Phi_0(\omega T)} + e^{j\Phi_1(\omega T)}] \qquad (2)$$

where $\Phi_0(\omega T)$ and $\Phi_1(\omega T)$ are the phase responses of $H_0(z)$ and $H_1(z)$, respectively. The magnitude of the overall filter is thus limited by

$$\left|H(e^{j\omega T})\right| \leq 1 \qquad (3)$$

The transfer function of a LWDF and its complementary transfer function are power complementary, *i.e.*,

$$\left|H(e^{j\omega T})\right|^2 + \left|H_c(e^{j\omega T})\right|^2 = 1 \qquad (4)$$

where

$$H_c(z) = \frac{1}{2} [H_0(z) – H_1(z)] \qquad (5)$$

This means that, if H(z), for example, is a low-pass filter, then a high-pass filter $H_c(z)$ can be obtained by simply changing the sign of the all-pass filter $H_1(z)$ in (1). It is known that, an attenuation zero exists corresponding to an angle $\omega_0 T$ at which the magnitude function reaches its maximum value. For LWDFs, this occurs when [21]

$$\left|H(e^{j\omega_0 T})\right| = 1 \qquad (6)$$

A transmission zero exists corresponding to an angle $\omega_1 T$ at which the magnitude function is zero, *i.e.* when

$$\left|H(e^{j\omega_1 T})\right| = 0 \qquad (7)$$

At an attenuation zero, the phase responses of the branches must take the same value. Hence, in the pass-band of the filter, the phase responses must be approximately equal , *i.e.*

$$\Phi_0(\omega T) = \Phi_1(\omega T) \qquad (8)$$

while, at a transmission zero, the difference in phase between the two branches must be

$$\Phi_0(\omega T) – \Phi_1(\omega T) = \pm\pi \qquad (9)$$

Thus, the difference in phase between the two branches must approximate $\pm\pi$ in the stop-band of the filter. To make sure that only one pass-band and one stop-band occur, the orders of $H_0(z)$ and $H_1(z)$ must differ by one [18, 21].

3. Linear Phase LWDFs

It is possible to obtain a LWDF (and more generally, a filter composed of two all-pass filters in parallel) with approximately linear phase by letting one of the branches consist of a pure M^{th} orders delay [21 – 23]. A linear phase LWDF is shown in Fig. 3. The low-pass transfer function of a linear-phase LWDF is

$$H(z) = \frac{1}{2} [H_0(z) + z^{-M}] \qquad (10)$$

The transfer function $H_0(z)$ corresponds to a general allpass function and can consequently be written as

$$H_0(z) = \frac{\sum_{i=0}^{N} b_n z^{i}}{\sum_{i=0}^{N} b_n z^{N-i}} \qquad (11)$$

For low-pass and high-pass filters N and M must be selected such that $N = M \pm 1$. The selection $N = M + 1$ gives the best result in most cases. Using such selection, the overall frequency response will be of the odd order type and can be expressed as

$$H(e^{j\omega T}) = \frac{1}{2}\left[e^{j\Phi_0(\omega T)} + e^{-M\omega T} \right] \qquad (12)$$

In the pass-band, the phase response of $H_0(e^{j\omega T})$ branch, $\Phi_0(\omega T)$, must approximate the phase response of the other branch which in this case is linear, *i. e.*, $\Phi_0(\omega T) = - M\omega T$. This forces the overall phase response to be approximately linear in the passband.

There exist no closed form solutions for the design of linear phase LWDFs, therefore, numerical optimization algorithms have be used. Many of these optimization approaches are reported for the design of almost linear phase LWDFs [21 -23]. In the next sections, a simple analytic procedure for the design of almost linear phase LWDFs is introduced with some illustrative examples.

4. The Design Procedure

The design of low-pass linear phase LWDF is considered here with cutoff frequency $\omega_c T$ (or ω_c for normalized case, *i.e.*, $T = 1$). The design procedure starts from (12) by approximately equating the phases of the two (all-pass and pure delay) branches of Fig. 3 in the pass-band as

$$\Phi_0(\omega T) = - M\omega T \pm \Delta \ , \quad \textit{for} \ \ (0 \le \omega T \le \omega_c T) \qquad (13)$$

and with approximately **- π** difference in the stop-band as

$$\Phi_0(\omega T) = - M\omega T - \pi \pm \Delta \ , \quad \textit{for} \ \ (\omega_c T \le \omega T \le \pi) \qquad (14)$$

where Δ is a small real value approaches zero at both attenuation and transmission zeros. The all-pass branch takes the following transfer function:

$$H_0(z^{-1}) = \frac{\sum_{n=0}^{N} b_n z^{-n}}{\sum_{n=0}^{N} b_n z^{-N+n}} \qquad (15)$$

where N=M+1 is the order of the all-pass section and b_n's are the coefficients to be determined.

From (15), one can write

$$H_0(e^{-j\omega T}) = \frac{\sum_{n=0}^{N} b_n e^{-jn\omega T}}{\sum_{n=0}^{N} b_n e^{-j(N-n)\omega T}} \qquad (16)$$

or

$$H_0(e^{-j\omega T}) = \frac{\sum_{n=0}^{N} b_n (\cos n\omega T - j \sin n\omega T)}{\sum_{n=0}^{N} b_n [\cos(N-n)\omega T - j \sin(N-n)\omega T]} \qquad (17)$$

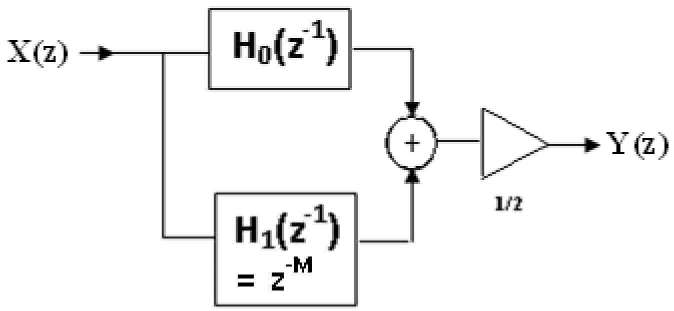


Fig. 3 Linear phase LWDF block diagram.

Thus, the phase response of the allpass branch is

$$\Phi_0(\omega T)= \tan^{-1}\left(\frac{\sum_{n=0}^{N} bn \ \sin n\omega T}{\sum_{n=0}^{N} bn \ \cos n\omega T}\right) - \tan^{-1}\left(\frac{\sum_{n=0}^{N} bn \ \sin (N-n)\omega T}{\sum_{n=0}^{N} bn \ \cos (N-n)\omega T}\right) \qquad (18)$$

From all-pass characteristic, it is known that

$$\tan^{-1}\left(\frac{\sum_{n=0}^{N} bn \ \sin n\omega T}{\sum_{n=0}^{N} bn \ \cos n\omega T}\right) = -\tan^{-1}\left(\frac{\sum_{n=0}^{N} bn \ \sin (N-n)\omega T}{\sum_{n=0}^{N} bn \ \cos (N-n)\omega T}\right)$$

Thus,

$$\Phi_0(\omega T) = 2\tan^{-1}\left(\frac{\sum_{n=0}^{N} bn \ \sin n\omega T}{\sum_{n=0}^{N} bn \ \cos n\omega T}\right) \qquad (19)$$

or

$$\sum_{n=0}^{N} bn \ \sin n\omega T = \tan\left(\frac{\Phi_0(\omega T)}{2}\right) \sum_{n=0}^{N} bn \ \cos n\omega T$$

i.e.,

$$\sum_{n=0}^{N} bn \left[\sin n\omega T - \tan\left(\frac{\Phi_0(\omega T)}{2}\right) \cos n\omega T \right] = 0 \qquad (20)$$

According to (13) and (14), (20) can be formulated in the pass-band as follows:

$$\sum_{n=0}^{N} bn \left[\sin n\omega T - \tan\left(\frac{-M\omega T}{2}\right) \cos n\omega T\right] = \pm\delta, \ \textit{for} \ (0 \le \omega T \le \omega_c T) \qquad (21)$$

and in the stop-band as follows:

$$\sum_{n=0}^{N} bn \left[\sin n\omega T - \tan\left(\frac{-M\omega T-\pi}{2}\right) \cos n\omega T\right] = \pm\delta, \ \textit{for} \ (\omega_c T \le \omega T \le \pi) \qquad (22)$$

where $\delta \ll 1$.

By selecting (N+1) extremal points on the union of the pass-band and the stop-band regions. Therefore, (21) and (22) are sampled in these frequency points, while proper alternating $\pm\delta$ values are examined at these points. One can start with an initial point $\omega_1 T > 0$ and the other points in the pass-band and stop-band can then be distributed equidistantly in the rest band. In matrix form, we can write the sampled version of (21) and (22) as

$$\mathbf{A}\ \mathbf{B} = \boldsymbol{\delta} \qquad (23)$$

where $\mathbf{A}$ is an (N+1) x (N+1) Matrix given by

$$\mathbf{A} = [\, a_{i\,j}],\ \textit{for}\ (i , j) = 1, 2, 3,. \ .(N+1) \qquad (24)$$

$\mathbf{B}$ is an (N+1) x 1 matrix can be written in a transposed form as

$$\mathbf{B}^t = [\ b_0 \ \ b_1 \ \ b_2 \ \ . \ . \ . \ \ b_N\ ] \qquad (25)$$

and $\boldsymbol{\delta}$ an **(N+1) x 1** matrix can be written in a transposed form as

$$\boldsymbol{\delta}^t = [\ \delta \ \ -\delta \ \ \ \delta \ \ -\delta \ . \ . \ . \ \ \delta\ ] \qquad (26)$$

with

$$a_{i\,j} = \sin 2(j-1)\omega_i T - \tan\left(\frac{-M\omega_i T}{2}\right) \cos 2(j-1)\omega_i T \qquad (27)$$

in the passband ($0 < \omega_i T \le \omega_c T$), i =1, 2, 3,…., N/2 (N even) [or i =1, 2, 3, ….,(N+1)/2 (N odd)] and j =1, 2, 3,….., N+1. Also

$$a_{ij} = \sin 2(j-1)\omega_i T - \tan\left(\frac{-M\omega_i T-\pi}{2}\right)\cos 2(j-1)\omega_i T \qquad (28)$$

in the stop-band ($\omega_c T < \omega_i T \le \pi$), i = (N/2) + 1, (N/2) + 2, ….., N+1 (N even) [or i = [(N+1)/2] +1, [(N+1)/2] + 2, ….., N+1 (N odd)] and j = 1, 2, 3….., N+1.

In this analytic algorithm, δ can be selected properly to solve (23) as

$$\mathbf{B} = \mathbf{A}^{-1}\ \boldsymbol{\delta} \qquad (29)$$

The design algorithm is thus reduced to the calculation of the vector

$\mathbf{B^t} = [b_0\ \ b_1\ \ b_2\ \ \ldots\ b_N]$ which represents all the b_n's coefficients that should appear in $H_0\ (z^{-1})$ of (15), while the total low-pass LWDF function $H_{LPF}\ (z^{-1})$ is the one given in (10). Polynomial factorization can be used to expand $H_0\ (z^{-1})$ of (15) into a product of 1st and 2nd order all-pass sections, while all the multiplier coefficients $\boldsymbol{\alpha_i}$ of the corresponding 1st and 2nd order adaptors in branch $H_0(z^{-1})$ can then be evaluated by using the same method given in [24]. It should be noted, here, that to design the corresponding high-pass complement filter, one can change the plus sign to minus in (10) to find the total high-pass function $H_{HPF}\ (z^{-1})$. It should also be noted that low-pass LWDF can be transformed to band-pass one $H_{BPF}\ (z^{-1})$ by setting each z in $H_{LPF}\ (z^{-1})$ equal to $-z^2$.

5. Design Examples

Five different examples are illustrated in this section to support the above design procedure of approximately linear phase of LWDFs, using the algorithm described in section 4. These examples are a 5th order LPF with cutoff frequency (0.34π), another 5th order HPF with cutoff frequency (0.34π), a 7th order LPF with cutoff frequency (0.5π), a 9th order LPF with cutoff frequency (0.58π), and 13th order LPF with cutoff frequency (0.42π). Applying the design procedure with $\boldsymbol{\delta} = 0.01$, the upper all-pass branches in the LWDF of Fig. 3 will be of a 3rd order type (N = 3 *i. e.*, M = 2) for the 5th order LPF, another 3rd order type (N = 3 *i. e.*, M = 2) for the 5th order HPF, a 4th order type (N = 4 *i. e.*, M = 3) for the 7th order LPF, a 5th order type (N = 5 *i. e.*, M = 4) for the 9th order LPF, and a 7th order type (N = 7 *i. e.*, M = 6) for the 13th order LPF. The resulting $H_o(z^{-1})$ and $H_1(z^{-1})$ with the total low pass or high pass responses, ($H_{LPF}(z^{-1})$ or $H_{HPF}(z^{-1})$), those correspond to the five examples are given in Table-1. The magnitude and phase responses are shown in Figs. 4 - 8.

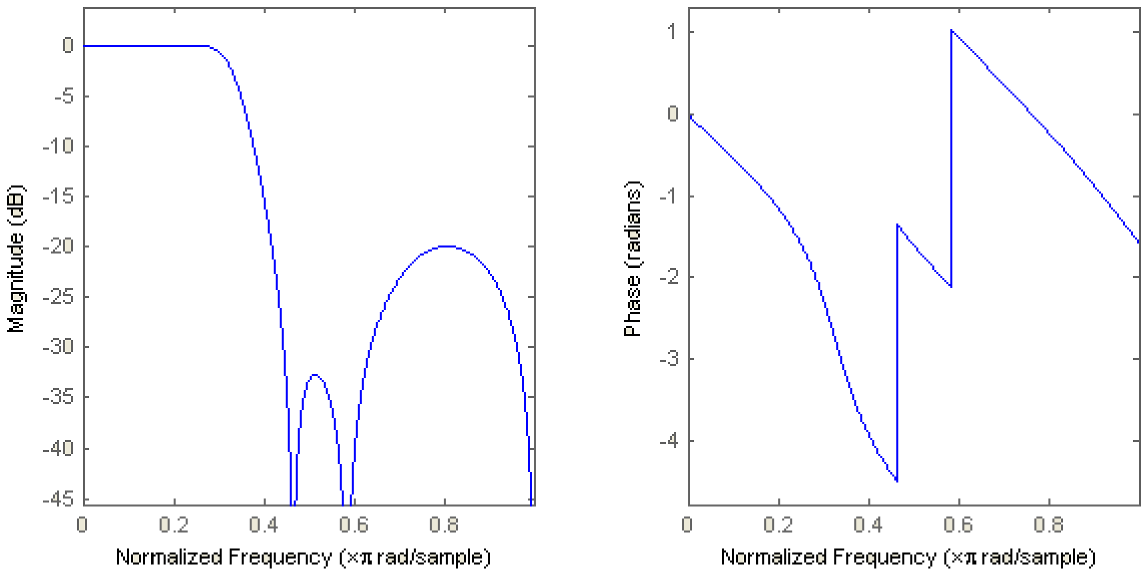
6. Conclusions

A simple design of an odd order almost linear phase LWDFs has been presented in this paper. Linear phase responses can approximately be achieved for these structures by replacing one of the all-pass branches of the original structures by a delay unit. Since there exist no closed form solutions for the design of linear phase LWDFs, therefore, numerical optimization algorithms have always been used. A simple analytic design procedure of almost linear phase LWDFs has been presented in this paper with some examples.

It has been noticed that the magnitude and phase responses of the designed filters give better representations of the desired ones as the order of the LWDF increases (about 90% of the pass-band preserve the linear phase property for filters with orders 9 and 13). That means more implementation complexity is required. this complexity may be reduced since all numerator polynomials of the total responses in Table-1 are image mirror ones. In addition to that, half-band LWDFs, such as the example of 7th order LPF, preserves the property of having some zero parameters in the total response which will provide more reductions in implementation. Fortunately, Such implementation may easily find its place on a single chip using bit-serial, bit-parallel processing elements or FPGA structures. It is promising topics to use such orthonormal structures (with half-band filters) in the wavelet transform implementations on a single FPGA chip or to use them as wavelet bases with perfect reconstruction Quadrature Mirror Filter (QMF) banks.

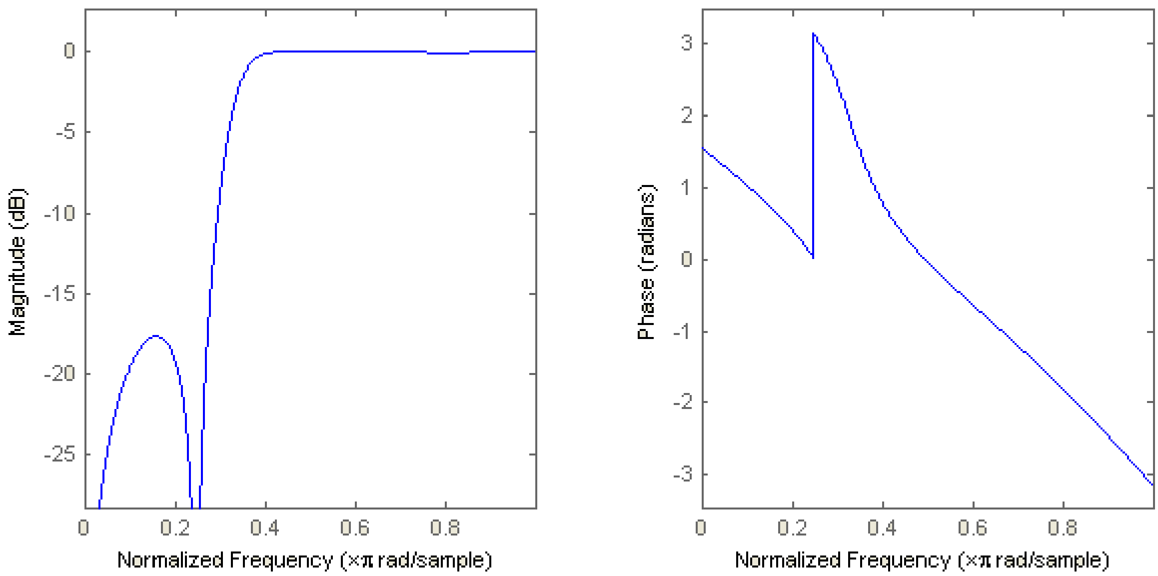
Table-1 The resulting $H_o(z^{-1})$ and $H_1(z^{-1})$ with the total responses ($H_{LPF}(z^{-1})$ or $H_{HPF}(z^{-1})$) according to LWDF type

Type of approximately linear phase LWDF	$H_o(z^{-1})$	$H_1(z^{-1})$	$H_{LPF}(z^{-1})$ or $H_{HPF}(z^{-1})$
5th order LPF with cutoff frequency (0.34π)	$\frac{0.2552+0.3350 z^{-1}-0.4390 z^{-2}+z^{-3}}{1-0.4390 z^{-1}+0.3350 z^{--2}+0.2552 z^{-3}}$	z^{-2}	$\frac{0.1276+0.1675 z^{-1}+0.2805 z^{-2}+0.2805 z^{-3}+0.1675 z^{-4}+0.1276 z^{-5}}{1-0.4390 z^{-1}+0.3350 z^{--2}+0.2552 z^{-3}}$
5th order HPF with cutoff frequency (0.34π)	$\frac{0.2552+0.3350 z^{-1}-0.4390 z^{-2}+z^{-3}}{1-0.4390 z^{-1}+0.3350 z^{--2}+0.2552 z^{-3}}$	z^{-2}	$\frac{0.1276+0.1675 z^{-1}-0.7195 z^{-2}+0.7195 z^{-3}-0.1675 z^{-4}-0.1276 z^{-5}}{1-0.4390 z^{-1}+0.3350 z^{-2}+0.2552 z^{-3}}$
7th order LPF with cutoff frequency (0.5π)	$\frac{-0.4608+0.3377 z^{-2}+z^{-4}}{1+0.3377 z^{-2}-0.4608 z^{-4}}$	z^{-3}	$\frac{0.2304+0.1689 z^{-2}+0.5 z^{-3}+0.5 z^{-4}+0.1689 z^{-5}-0.2304 z^{-7}}{1+0.3377 z^{-2}-0.4608 z^{-4}}$
9th order LPF with cutoff frequency (0.58π)	$\frac{0.0944-0.1751 z^{-1}+0.0006 z^{-2}+0.5488 z^{-3}+0.3700 z^{-4}+z^{-5}}{1+0.3700 z^{-1}+0.5488 z^{-2}+0.0006 z^{-3}-0.1751 z^{-4}+0.0944 z^{-5}}$	z^{-4}	$\frac{0.0472-0.0875 z^{-1}+0.0003 z^{-2}+0.2744 z^{-3}+0.6850 z^{-4}+0.6850 z^{-5}+0.2744 z^{-6}+0.0003 z^{-7}-0.0875 z^{-8}+0.0472 z^{-9}}{1+0.3700 z^{-1}+0.5488 z^{-2}+0.0006 z^{-3}-0.1751 z^{-4}+0.0944 z^{-5}}$
13th order LPF with cutoff frequency (0.42π)	$\frac{0.0519-0.0323 z^{-1}-0.0328 z^{-2}-0.0948 z^{-3}-0.0402 z^{-4}+0.4218 z^{-5}-0.2319 z^{-6}+z^{-7}}{1-0.2319 z^{-1}+0.4218 z^{-2}-0.0402 z^{-3}-0.0948 z^{-4}-0.0328 z^{-5}-0.0323 z^{-6}+0.0519 z^{-7}}$	z^{-6}	$\frac{0.0260-0.0162 z^{-1}-0.0164 z^{-2}-0.0474 z^{-3}-0.0201 z^{-4}+0.2109 z^{-5}+0.3840 z^{-6}+0.3840 z^{-7}+0.2109 z^{-8}-0.0201 z^{-9}-0.0474 z^{-10}-0.0164 z^{-11}-0.0162 z^{-12}+0.0260 z^{-13}}{1-0.2319 z^{-1}+0.4218 z^{-2}-0.0402 z^{-3}-0.0948 z^{-4}-0.0328 z^{-5}-0.0323 z^{-6}+0.0519 z^{-7}}$



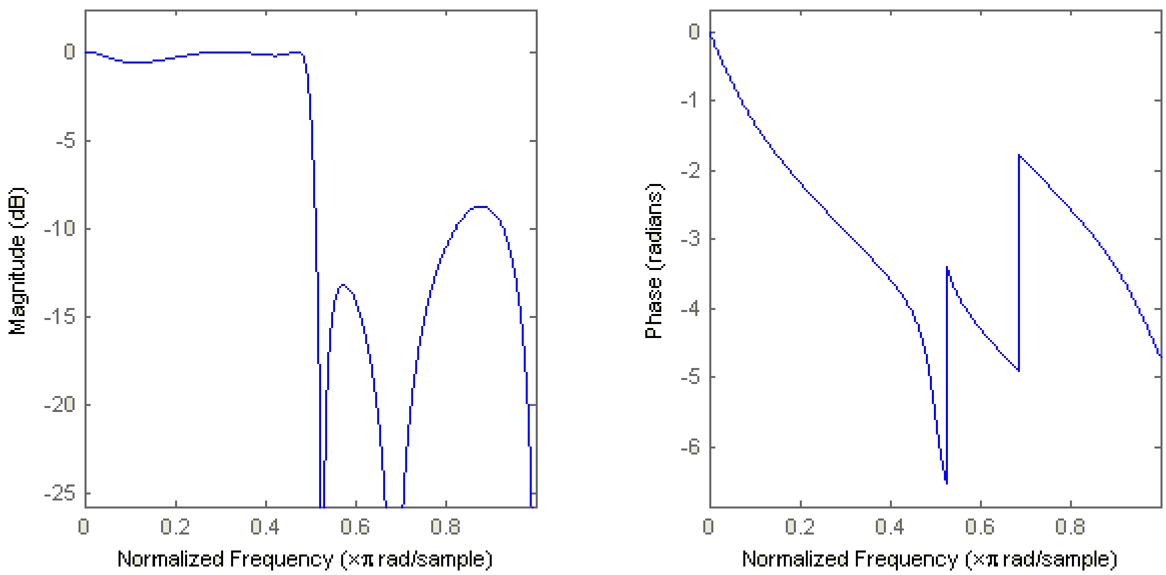
(a) (b)

Fig. 4 Responses of the 5th order LPF; (a) Magnitude, (b) Phase.



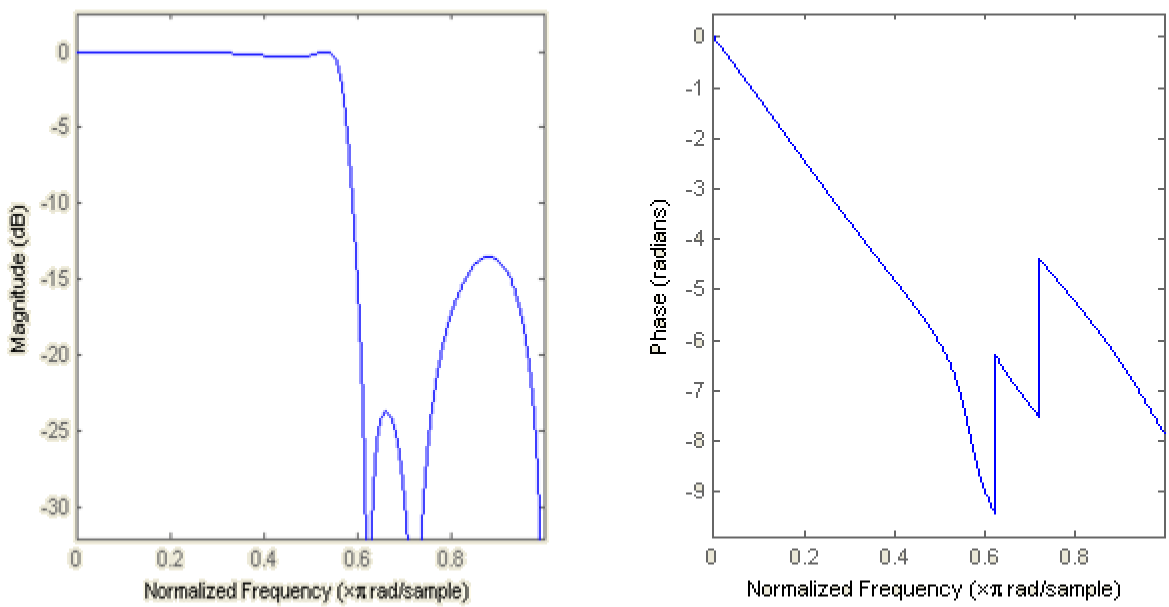
(a) (b)

Fig. 5 Responses of the 5th order HPF; (a) Magnitude, (b) Phase.



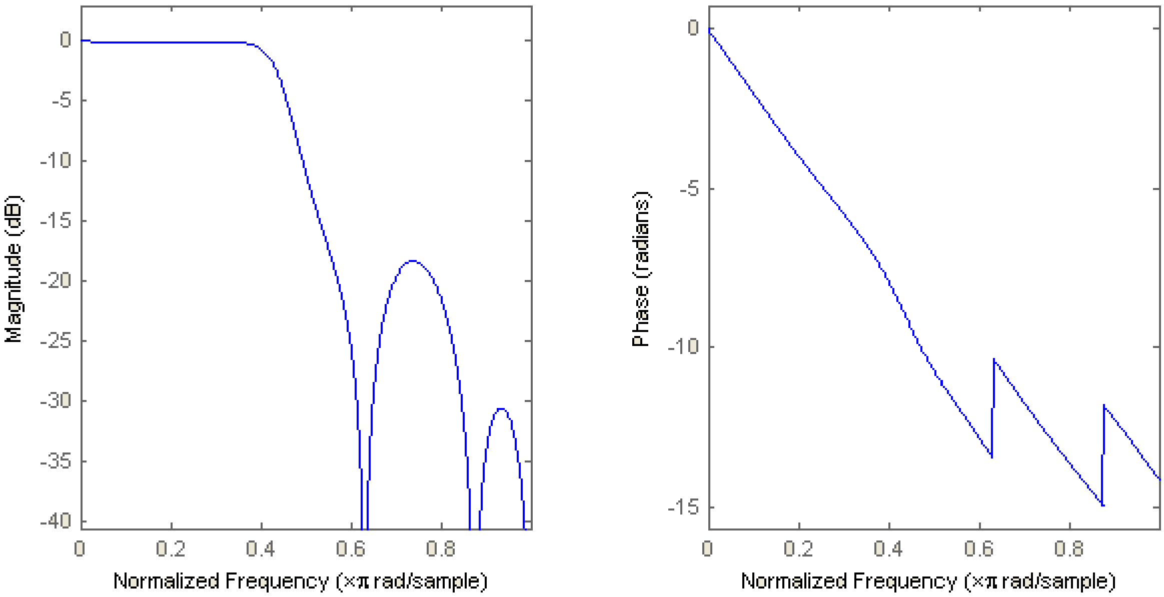
(a) (b)

Fig. 6 Responses of the 7th order LPF; (a) Magnitude, (b) Phase.



(a) (b)

Fig. 7 Responses of the 9th order LPF; (a) Magnitude, (b) Phase.



(a) (b)

Fig. 8 Responses of the 13th order LPF; (a) Magnitude, (b) Phase.

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